

Hypercrystalline vacua

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February 1, 2008

Abstract

Let a quantum network be a Fermi-Dirac assembly of Fermi-Dirac assemblies of ...of quantum points, interpreted topologically. The simplest quantum-network vacuum modes with exact conservation of relativistic energy-momentum and angular momentum also support the local non-Abelian groups of the standard model, torsion and gravity. The spacetime points of these models naturally obey parastatistics. We construct a left-handed vacuum network among others.

1 Vacua as condensates

We work at a level deeper than field theory, where both spacetime and fields resolve into a topological network of points and links, subject to strong locality and q (quantum) superposition. A vacuum network is a mode (vector) of high symmetry, a q four-dimensional crystal, or “hypercrystal”. Physical continua resolve into q superpositions of discrete modes; classical spacetime coordinates and fields are parameters of coherent states. Unlike Newton’s crystalline ether, the vacuum hypercrystal is Poincaré invariant due to q superposition. The gauge and matter fields are not extrinsic occupants but intrinsic excitations or defects of this q spacetime network, to which the hypercrystal is transparent or even superconducting. Meissner-Higgs-type effects concentrate certain imperfections into two-dimensional sheets, the flux sheets of gravity and the other gauge forces of present-day physics. The standard-model symmetries are thus keys to the hypercrystallography of the vacuum.

The initial results are unexpectedly simple. Any q hypercubic lattice is Poincaré invariant and has cell groups corresponding to the standard model,

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torsion and gravity and no other non-abelian gauges. Thus q topological kinematics and dynamics may turn out to account for all the forces rather naturally.

Specifically, we assume here that the (spacetime-matter) network is a set of “links”, which are sets of points. This is simply a “third-quantized” Fermi-Dirac assembly of Fermi-Dirac assemblies. The kets of q spacetime points, not analyzed further at this stage, form a linear space \mathcal{P} of high dimension $|\mathcal{P}| \rightarrow \infty$. We posit nearest neighbor interactions only.

General covariance was not truly general, but respected differentiability. A truly-general covariance permits arbitrary point-permutations. We posit a *quantum covariance* under $SL(\mathcal{P})$, which includes these as basis permutations. This activates the topology as Einstein’s less-general covariance activated the metric. Metric variations accounted for gravity; we require topological ones to account for all the forces.

The “spacetime code” and other early forms of quantum network dynamics (qnd)¹ fixed the cell structure at the start. This violates q covariance. The “cell” of a manifold is infinitesimal and is respected by diffeomorphisms but the qnd cell is finite. The qnd dynamics must govern the spacetime cell structure too, just as one dynamics governs atoms, molecules and crystals. Such a multilevel dynamics calls for a multilevel tensor analysis. We summarize next the one used here.

2 Metatensors and parastatistics

Metatensors are tensors over tensors over We use them where a classical theory uses sets of sets of Assume Fermi-Dirac statistics, generalizations being clear. The quantum algebra $\mathbf{Q}\mathcal{V}$ over any vector space \mathcal{V} with dual \mathcal{V}^\dagger is the Clifford algebra over the normed linear space $\mathcal{V} \oplus \mathcal{V}^\dagger$ with norm $\|v \oplus \omega\| := \omega(v)$.² If \mathcal{V} is the ket space of a system S then its quantum algebra consists of the linear operators of an (F-D) assembly of S ’s. By the *metaquantum* algebra over \mathcal{V} we mean $\mathcal{Q}\mathcal{V} := \lim_{L \rightarrow \infty} \mathbf{Q}^L \mathcal{V}$. Its elements we call metatensors over \mathcal{V} . The hypothetical q system with ket space $\mathcal{Q}\mathcal{V}$ we call the universal quantum over \mathcal{V} . It is self-referential in much the way that set theory is: $\mathcal{Q}\mathcal{V}$ includes all (sufficiently finite) operators on itself.

Tensors have grade g . Metatensors have level L , counting unitizations up from $\mathcal{V} \oplus \mathcal{V}^\dagger$ and a grade g^L at each level. They also have constituents at any depth $D \leq L$ counting down from L and a grade g_D at each depth.

Here we take the qnd network to have as its quantum algebra the universal quantum algebra $\mathcal{Q} := \mathcal{Q}\{0\}$ and basis $N_Q = |Q\rangle$. Then qnd kets are first-grade elements of \mathcal{Q} . Link kets in turn are depth-one grade-one factors of qnd kets. Spacetime point kets are depth-two grade-one factors of qnd kets. For

¹Finkelstein, D. (1996). *Quantum Relativity*. Springer, Heidelberg. And references cited there.

² H. Saller, Quantum algebras I, II. *Il Nuovo Cimento* **108B**, 603, **109B**, 255.

any depth $D \in \mathbb{N}$, let c_Q^D be the creation operator on \mathcal{Q} creating factors $|Q\rangle$ at depth D with dual annihilator ∂_D^Q . We identify quantum covariance with invariance under the group generated by all $L^Q_P c_Q^1 \partial_1^P$ with $\text{tr } L = 0$.

The action operator \tilde{S} acts on \mathcal{Q} , hence $\tilde{S} \in \mathcal{Q}(\mathcal{P})$. Qnd kets Ψ (e.g., $|\text{vac}\rangle$) must obey the differential subsidiary conditions $[\partial_Q^1 \tilde{S}] \Psi = 0$.

Typical action terms:

- $c_A \partial^A = \text{network number, modulus}$
- $c_Q^1 \partial_1^Q \sim \text{link number}$
- $c_{2Q} \partial_1^Q \sim \text{point number}$
- $c_Q^2 c^1[\mathcal{Q}_P] \partial_2^P \dots \sim \text{first-neighbor point-interaction}$

with composite link index $[\mathcal{Q}_P]$.

Points obey parastatistics. This is an unintended immediate consequence of qnd, not a separate physical hypothesis. A product extensor in $\mathbf{Q}^2(\mathcal{P})$ must change sign when two of its (first-grade!) factors are interchanged, but need not change sign when two subfactors of its factors are interchanged. This simple quantum fact has a simple classical analogue: Any set (for example, $\{\{1, 2\}, \{3, 4\}\}$) is invariant under an interchange of two elements (say, $\{1, 2\}$ and $\{3, 4\}$), but not under the interchange of *their* elements say, 1 and 3).

3 Metrical vacua

We turn now to empirical hypercrystallography of the vacuum. Assume four commuting coordinate operators n^μ on a point-ket space \mathcal{P} , each with spectrum \mathbb{Z} (not \mathbb{N} as in earlier work¹), defining a hypercubical array of eigenkets

$$|n^1, n^2, n^3, n^4\rangle = |n\rangle \in \mathcal{P}, \quad \langle n^1, n^2, n^3, n^4| = \langle n| \in \mathcal{P}^\dagger. \quad (1)$$

These are not fundamental but descend from the higher level of dynamics by spontaneous $\text{SL}(\mathcal{P})$ -breaking, and give rise in turn to the still-less-fundamental points and coordinates of Minkowski spacetime as follows. We identify the spacetime translation generators ∂_μ with down-shift operators (not antihermitian!) and the point coordinate operators with upshift operators,

$$\begin{aligned} \partial_\mu &:= \sum_n |n - 1_\mu\rangle \vee \langle n| \quad , \\ x^\mu &:= \sum_n |n + 1_\mu\rangle \vee \langle n| \quad (n + 1_\mu), \end{aligned} \quad (2)$$

scaled to save the commutation relations of differential geometry, $[\partial_\mu, x^\lambda] = \delta_\mu^\lambda$. It is then easy to see that q covariance includes general covariance, $\text{SL}(\mathcal{P}) \supset$

$\text{Diff}(\mathbb{R}^4) \supset \text{ISO}(1, 3)$. Namely, the representative $\delta\Lambda : \mathcal{P} \rightarrow \mathcal{P}$ of the infinitesimal diffeomorphism $\delta x = (\delta x(x)) : \mathbb{R}^4 \rightarrow \mathbb{R}^4$ is $\delta\Lambda = -\delta x \cdot \partial$. To represent $\text{POINCARÉ} = \text{ISO}(1, 3)$, one sets $\delta x^\mu = \alpha^\mu + \omega^\mu{}_\nu x^\nu$ with infinitesimal parameters α, ω with $\omega_{\mu\nu} = -\omega_{\nu\mu}$.

We propose some suitably symmetric patterns of topological linkages among these point-kets for vacuum hypercrystals. One early¹ model was a four-dipole metatensor with off-diagonal long-range order, $|\text{vac}\rangle = \iota\partial_1 \vee \dots \vee \iota\partial_4$. Like a hypervolume element, this is clearly invariant under the above non-unitary Poincaré subgroup of Diff . Its unit cell has $4! = 2 \times 3 \times 4$ bonus discrete symmetries that we tentatively identified with generators of quark hypercharge, isospin, color and spin: mutually-commuting coherent-state $U_1, \text{SU}_2, \text{SU}_3$, and Spin_4 groups.¹

This vacuum, however, is *too* symmetric. Like a hypervolume element, it is invariant under the inhomogeneous special linear group $\text{ISL}(4) \supset \text{ISO}(1, 3)$. It defines the volume element but not the metric. We construct three metrical vacua here.

The “Dalembertian vacuum” is the metatensor³

$$|\text{vac}\rangle \sim \iota g^{\mu\nu} \iota[\partial_\mu \circ \partial_\nu]. \quad (3)$$

The $4!$ discrete symmetries respect this for $g^{\mu\nu} = 1 - \delta^{\mu\nu}$ (null symmetric form). The “Dirac vacuum” is a square root of the Dalembertian vacuum:

$$|\text{vac}\rangle \sim \iota[\gamma^\mu \vee \iota\partial_\mu], \quad (4)$$

The “left-handed vacuum” is a restriction of the Dirac vacuum:

$$|\text{vac}\rangle \sim \iota[\sigma^\mu \vee \iota\partial_\mu] \quad (5)$$

(3) uses the ∂ given by (2). (3) uses the operator product \circ because the Grassmann product would give 0. (4) uses a Kähler representation of the Dirac γ operators by network metatensors defined elsewhere¹. (5) uses a γ^5 superselection rule, Hestenes’ identification $\gamma^5 = i$, and $\sigma^\mu \sim \gamma^\mu$. All three vacua have the local standard-model unitary symmetries and global proper Poincaré symmetries; only the left-handed vacuum lacks parity.

These quasi-empirical hypercrystalline vacua conserve angular momentum and energy-momentum but not shear and dilation. Their spacetime points automatically exhibit parastatistics. Their cells have algebras isomorphic to the local algebras of gravity, torsion and the standard model. One violates parity. The chances of one of these guessed vacua being right are nevertheless slim. They simply show that if we take quantum theory seriously, we can make quite simple locally finite hypercrystalline vacua that have the same symmetries as the singular classical continua used in the usual quantum field theory, which arbitrarily restricts quantum theory to one level. We must successively correct the vacuum structure and the action in turn to converge to a working theory.

³This improves on an earlier “quadrupole vacuum” of D. Finkelstein, H. Saller, and Z. Tang, Beneath gauge, *Nuclear Physics*, to appear in the Trautman issue.